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Semi-empirical systematics of (n, p) reaction cross-section at 14.5, 20, and 30 MeV

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Abstract

A new semi-empirical formula for the evaluation of the (n, p) reaction cross-section is discussed. The formula was derived using analytical expressions for the calculation of the proton emission spectrum in nuclear reactions within the framework of the preequilibrium exciton model and the evaporation model. A new analysis of experimental (n, p) reaction cross-sections was performed. The experimental data and model calculations were used to get the (n, p) reaction cross-section systematics at 14.5, 20, and 30 MeV. The systematics provides the best description of experimental data compared with formulas proposed earlier by other authors.

1. Introduction

A systematics is used for the evaluation of neutron induced reaction cross-section, when experimental data are scarce and results of model calculations seem unreliable.

Various approaches are applied to study the systematic dependence of neutron induced reaction cross-sections. As a rule, in empirical approaches, the cross-section is expressed by an exponential function with its argument depending on the number of nucleons in the target nucleus [1]. The formal use of the evaporation model for the justification of the exponential dependence of the cross-section, as it was done in many papers, does not seem correct, because it ignores an important contribution of the non-equilibrium particle emission in the reaction cross-section for medium and heavy nuclei. The semi-empirical approach to the study of the systematic depen-

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dence of neutron induced reaction cross-sections with detailed description of the non-equilibrium and equilibrium particle emission has been suggested in Refs. [2–9]. It was shown that the use of the method [4–9] results in a better description of experimental data (minimal χ^2 values) than can be obtained from the best empirical approaches. Further development of the approach [4–9] is done in this work.

The quality of the systematics depends also on the data set used to get numerical values of free parameters. The use of incomplete and out-dated sets of experimental data, as subjective criteria for the selection of measurements, as it is sometimes done, reduces the value of systematics, as independent and reliable instrument for the cross-section prediction.

A new semi-empirical formula for the evaluation of the (n, p) reaction cross-section is suggested in this paper. The formula was obtained using analytical expressions describing the contribution of non-equilibrium and equilibrium proton emission in the (n, p) reaction cross-section. The new analysis of available experimental data at 14.5 MeV has been performed for target nuclei with the mass number from 40 to 209. The formula provides the best fit for experimental data at 14.5 MeV compared with systematics proposed by other authors. The new systematics was obtained also for the 20 and 30 MeV—neutron incident energy.

2. Basic principles of the development of the (n, p) reaction cross-section systematics

2.1. The preequilibrium proton emission

According to the preequilibrium exciton model in a "closed" form [10] the energy distribution of protons emitted from the nucleus before the attainment of an equilibrium state is equal to

$$\frac{d\sigma^{\mathrm{pre}}}{d\varepsilon_p} = \sigma_{\mathrm{non}}(E_n)\frac{(2S_p+1)\mu_p\varepsilon_p\sigma_p^{\mathrm{inv}}(\varepsilon_p)}{\pi^2\hbar^3} \times \sum_{n=n_0} R_p(n)\frac{\omega(p-1,h,U_p)}{\omega(p,h,E_0)}\frac{1}{\lambda_n^+ + \lambda_n^- + \gamma_n}D(n), \tag{1}$$

where σ_{non} is the cross-section of non-elastic interaction of the incident neutron with a nucleus at the kinetic energy E_n; S_p and μ_p are spin and reduced mass of the outgoing proton; ε_p is the kinetic energy of the proton; σ_p^{inv} is the inverse reaction cross-section for proton; $\omega(p, h, E)$ is the density of exciton states with p particles and h holes $(p + h = n)$ at the excitation energy E; U_p is the excitation energy of the nucleus formed after the proton escape, $U_p = \alpha E_n + Q_{(n,p)} - \varepsilon_p$ and $\alpha = A/(A + 1)$, $Q_{(n,p)}$ is the energy of the (n, p) reaction; $E_0 = \alpha E_n + Q_n$, where Q_n is the neutron separation energy for the compound nucleus; λ_n^+ and λ_n^- are transition rates from the n-exciton state to the states with $n + 2$ and $n - 2$ excitons, correspondingly; γ_n is the nucleon emission rate; $R_p(n)$ is the factor describing the difference between the number of neutrons and protons in the n-exciton state; $D(n)$ is the factor, which takes into account the "depletion" of the n-exciton state due to the nucleon emission; n_0 is the initial exciton number $(n_0 = 3)$.

Using the Strutinski–Ericson formula [11] for exciton level density, the Williams approach [12] for the calculation of λ_n^+ and assuming that the transition rate from n to $n + 2$ state visibly exceeds the rates λ_n^- and γ_n, the expression for proton emission spectrum can be written as follows

$$\frac{d\sigma^{\mathrm{pre}}}{d\varepsilon_p} = \sigma_{\mathrm{non}}(E_n)\frac{(2S_p+1)\mu_p\varepsilon_p\sigma_p^{\mathrm{inv}}(\varepsilon_p)}{2\pi^3\hbar^2 g^4 E_0^3|M|^2}\sum_{n=n_0} R_p(n)\left(\frac{U_p}{E_0}\right)^{n-2} p\left(n^2-1\right), \tag{2}$$

where $|M|^2$ is the mean square of the matrix element of the residual nuclear interaction; g is the single particle level density.

The inverse reaction cross-section can be evaluated using the "sharp cut-off" approximation: $\sigma_p^{\mathrm{inv}}(\varepsilon_p) = \pi R^2(1 - V_p/\varepsilon_p)$, where R is the nucleus radius and V_p is the Coulomb potential for protons. Assuming that the term $n = 3$ gives the main contribution in Eq. (2), after the integration from the minimal to maximal available energy for the proton emission $AE_n/(A+1) + Q_{(n,p)}$ one obtains the expression for the precompound component of the (n, p) reaction cross-section

$$\sigma_{(n,p)}^{\mathrm{pre}} = \sigma_{\mathrm{non}}(E_n)\frac{(2S_p+1)\mu_p R^2}{\pi^2\hbar^2 g^4 E_0^4 |M|^2}(\alpha E_n + Q_{(n,p)} - V_p)^3, \tag{3}$$

where $R_p(3)$ is taken equal to 0.75 [13]; V_p is the Coulomb potential for protons; R is the nucleus radius.

2.2. The equilibrium proton emission

Using the Weisskopf–Ewing theory [14] and assuming that the neutron emission width exceeds widths for other channels, the equilibrium component of the (n, p) reaction cross-section can be evaluated as follows (see, e.g., Refs. [4,15])

$$\sigma_{(n,p)}^{\mathrm{eq}} = \sigma_{\mathrm{non}}(E_n)\big(1 - P^{\mathrm{pre}}\big)\frac{\mu_p(2S_p+1)}{\mu_n(2S_n+1)}\exp\left[\frac{\alpha E_n + Q_{(n,p)} - V_p}{T_p} - \frac{\alpha E_n}{T_n}\right], \tag{4}$$

where P^{pre} is the total probability of non-equilibrium processes; S_n and μ_n are spin and reduced mass of the outgoing neutron; T_n and T_p are nuclear temperature for residuals, produced after the neutron and proton emission, correspondingly.

The nuclear temperature can be evaluated using the simple relation between the nuclear level density parameter and atomic mass number $a = A/C$, where C is a constant,

$$T_p \cong T_n = \sqrt{\alpha E_n/a}, \tag{5}$$

where it is assumed that E_n substantially exceeds the $Q_{(n,p)}$ value.

Using Eqs. (4) and (5) one can obtain the approximate expression for $\sigma_{(n,p)}^{\mathrm{eq}}$

$$\sigma_{(n,p)}^{\mathrm{eq}} = \sigma_{\mathrm{non}}\big(1 - P^{\mathrm{pre}}\big)\exp\left[\frac{A^{1/2}}{(C\alpha E_n)^{1/2}}(Q_{(n,p)} - V_p)\right]. \tag{6}$$

2.3. The general form of the systematics

According to the semi-empirical mass formula the energy of the (n, p) reaction is approximately equal to

$$Q_{(n,p)} = \beta_1\left(\frac{N - Z + 1}{A}\right) + \beta_2 Z/A^{1/3}, \tag{7}$$

where N, Z, and A are number of neutrons, protons, and nucleons in the target nucleus, correspondingly; β_i are constants.

All nuclei can be optionally divided in two big groups with the dominant contribution of equilibrium and preequilibrium proton emission in the (n, p) reaction cross-section at 14.5 MeV [16]. For the first group the systematics can been obtained using Eqs. (6)–(7). For nuclei with dominant contribution of the preequilibrium emission in the (n, p) reaction cross-section, the formula for the systematics can be derived from Eqs. (3) and (7). As the results the following 7-parametric formula is suggested for the evaluation of the cross-section for target nuclei with $Z \leqslant 50$

$$\sigma_{(n,p)} = \pi r_0^2 \big(A^{1/3} + 1\big)^2 \exp\big(A^{0.5}(\alpha_1 S + \alpha_2 V + \alpha_3)\big), \tag{8a}$$

for nuclei with $Z > 50$

$$\sigma_{(n,p)} = \pi r_0^2 \big(A^{1/3} + 1\big)^2 A^{\alpha_4} (\alpha_5 S + \alpha_6)^3, \tag{8b}$$

where $S = (N - Z + 1)/A$, $V = Z/A^{1/3}$, $r_0 = 1.3$ fm, α_i are parameters.

3. Systematics of the (n, p) reaction cross-section

3.1. Incident neutrons of energy 14.5 MeV

3.1.1. Experimental data

Experimental data for the (n, p) reaction were taken from EXFOR. The data available in the energy range 14–15 MeV were reduced to 14.5 MeV using excitation functions for the (n, p) reaction from Ref. [17].

The statistical treatment of experimental data available for a single nucleus has been performed using the method of "weighted mean" [18]

$$\big\langle \sigma^{\exp} \big\rangle = \frac{\sum_{j=1}^{n} \sigma_j / (\Delta\sigma_j)^2}{\sum_{j=1}^{n} 1/(\Delta\sigma_j)^2}, \qquad \big\langle \Delta\sigma^{\exp} \big\rangle = \max(\Delta\sigma_A, \Delta\sigma_B), \tag{9}$$

and

$$\Delta\sigma_A = \left(\frac{\sum_{j=1}^{n} (\sigma_j - \langle\sigma\rangle)^2 / (\Delta\sigma_j)^2}{(n-1) \sum_{j=1}^{n} 1/(\Delta\sigma_j)^2} \right)^{1/2}, \qquad \Delta\sigma_B = \left(\sum_{j=1}^{n} 1/(\Delta\sigma_j)^2 \right)^{-1/2}, \tag{10}$$

where $\langle \sigma^{\exp} \rangle$ and $\langle \Delta\sigma^{\exp} \rangle$ are the (n, p) reaction cross-section and its error evaluated using data of different measurements; σ_i and $\Delta\sigma_i$ is the cross-section and its error, obtained in ith experiment; n is the number of measurements performed for the nucleus,

The (n, p) reaction cross-section, $\langle \sigma^{\exp} \rangle \pm \langle \Delta\sigma^{\exp} \rangle$, obtained from the analysis of experimental data for 125 nuclei with $A \geqslant 39$, are shown in Table 1.

3.1.2. Comparison of different systematics

The most advanced systematics [6,19] are used for the comparison with the formula proposed, Eq. (8).

Belgaid and coauthors [19]:

$$\sigma_{(n,p)} = \big(A^{1/3} + 1\big)^2 \big[\exp\big(\alpha_1 + \alpha_2 S_1^2 + \alpha_3 S_2^2\big) + \big(\alpha_4 + \alpha_5 S_1^2 + \alpha_6 S_1\big)^3\big] \tag{11}$$

and Konobeyev, Korovin [6]:

$$\sigma_{(n,p)} = \pi r_0^2 \big(A^{1/3} + 1\big)^2 \big[A^{\alpha_1} \big(\alpha_2 S_1^2 + \alpha_3 S_1 + \alpha_4\big)^3 + \alpha_5 \exp\big(\alpha_6 S_1^2 + \alpha_7 S_1\big)\big], \tag{12}$$

where $S_1 = (N - Z + 1)/A$, $S_2 = (N - Z + 1)/A^{4/3}$.

Table 1
The (n, p) reaction cross-section at the incident neutron energy 14.5 MeV obtained from the analysis of experimental data ($\sigma_i^{\text{exp}} \pm \Delta\sigma_i^{\text{exp}}$), the cross-section calculated using Eq. (8) with parameters from Table 2 (σ_i^{syst}), the value of $\Sigma_i = ((\sigma_i^{\text{syst}} - \sigma_i^{\text{exp}})/\Delta\sigma_i^{\text{exp}})^2$ and accession numbers of the EXFOR files used for the analysis

Z	A	$\sigma_i^{\text{exp}} \pm \Delta\sigma_i^{\text{exp}}$ (mb)	σ_i^{syst} (mb)	Σ_i	EXFOR files			
18	40	17.1 ± 2.7	25.6	9.80	11585,	11554,	11548	
19	39	132 ± 28	230	12.1	21668,	40433,	13109,	21846
19	41	49.6 ± 5.0	58.5	3.16	20835,	12957,	30011,	22089,
					41240,	20811,	30263,	20889,
					21846,	21976,	30336,	30707,
					40223			
20	40	461 ± 50	517	1.24	21165,	21846		
20	42	176 ± 18	132	6.10	40016,	30011,	22089,	20198,
					21608,	21976,	30115,	31545,
					40223,	20090		
20	43	97.8 ± 9.8	67.9	9.29	30011,	22089,	20198,	21608,
					30115,	31545		
20	44	38.2 ± 3.8	35.6	0.465	40016,	30011,	22089,	22611,
					20721,	21608,	21846,	21976,
					30115,	31545,	40223,	20090
21	45	57.0 ± 5.7	78.6	14.4	11462,	31494,	30115,	40223
22	46	263 ± 26	171	12.5	22093,	20887,	30979,	32592,
					20926,	22089,	11631,	30523,
					21300,	11633,	20721,	31464,
					11630,	30660,	30825	
22	47	164 ± 16	90.7	21.0	22093,	12956,	20986,	13133,
					22089,	11610,	11631,	30825,
					20721,	31464,	30979,	11630,
					30660,	30810,	40226	
22	48	64.6 ± 6.5	48.7	5.95	22093,	12956,	20887,	20986,
					21941,	22214,	13133,	30523,
					32592,	30979,	11494,	11610,
					11631,	12977,	21300,	22089,
					20721,	20815,	20931,	31464,
					31496,	11630,	30336,	30660,
					30707,	30804,	30810,	30825,
					40226			
22	49	34.1 ± 3.4	26.5	4.97	11631,	20721,	11630,	30336,
					30707,	40226		
22	50	13.3 ± 1.3	14.6	0.997	22215,	22281,	22433,	21901,
					30648,	22089,	30978,	11631,
					20811,	11633,	21976,	11630,
					30336,	30660,	30707,	30810,
					40226			
23	51	26.7 ± 2.7	31.2	2.77	12956,	21343,	12769,	22156,
					22089,	22703,	20811,	21824,
					30263,	30562,	12069,	20721,
					20931,	21893,	21999,	30654,
					12969,	20799,	30013,	30336,
					30707,	40226,	22656	
24	52	81.9 ± 8.2	65.8	3.85	40433,	12958,	11464,	22187,
					22433,	11536,	22089,	30978,
					30263,	30812,	20673,	20721,
					31464,	11132,	11657,	30013,

Table 1 (*continued*)

Z	A	$\sigma_i^{exp} \pm \Delta\sigma_i^{exp}$ (mb)	σ_i^{syst} (mb)	Σ_i	EXFOR files			
					30336,	30707,	30810,	31161,
					41321			
24	53	42.6 ± 4.3	36.6	1.97	22187,	22433,	30978,	30812,
					20673,	20721,	21936,	21976,
					31464,	11132,	11657,	30336,
					30810,	41321,		
24	54	16.5 ± 1.6	20.5	6.34	22215,	22281,	22433,	30812,
					20673,	20721,	21936,	11657,
					30810,	41321		
25	55	32.2 ± 7.7	42.7	1.86	20903,	31039,	30013,	30336
26	54	323 ± 32	282	1.62	20396,	11721,	22093,	11696,
					20961,	21352,	22214,	41118,
					11722,	22089,	41240,	30671,
					30979,	10309,	12977,	10022,
					20841,	13586,	20554,	21936,
					31496,	31459,	41321	
26	56	110 ± 11	87.9	4.05	11703,	21923,	22093,	11696,
					10417,	11474,	12956,	20280,
					21352,	21487,	22214,	41118,
					11701,	30483,	10289,	11494,
					30562,	31479,	20377,	22089,
					22312,	41240,	20890,	30979,
					10309,	30993,	10022,	30755,
					20772,	20554,	20815,	20888,
					30676,	11715,	31524,	20798,
					10835,	11718,	20887,	21372,
					30707,	30802,	30807,	40223,
					41313,	21419		
26	57	53.5 ± 7.0	49.8	0.286	21352,	22325,	10309,	11696,
					30823,	20721,	21936,	30978,
					11718			
26	58	13.1 ± 4.8	28.4	10.2	41320,	21936,	30978	
27	59	48.7 ± 4.9	57.8	3.46	30805,	31449,	31520,	12929,
					22132,	22327,	11492,	22089,
					41240,	30979,	12977,	31039,
					21973,	20841,	30263,	32591,
					40136,	21936,	31464,	12969,
					11740,	11741,	31459,	40223
28	58	318 ± 32	360	1.72	20396,	31412,	31500,	22093,
					11696,	32585,	32593,	10836,
					20898,	20986,	21487,	31038,
					41118,	20303,	30985,	11761,
					10484,	30562,	22089,	32592,
					30604,	31455,	30979,	41240,
					31107,	12977,	10022,	20841,
					30263,	40136,	20527,	21936,
					31524,	11740,	22618,	30811,
					31459,	40226,	22820	
28	60	142 ± 14	116	3.37	20390,	32593,	31495,	30979,
					40136,	32577,	20388,	22637,
					41240,	11696,	12977,	30825,

(*continued on next page*)

Table 1 (*continued*)

Z	A	$\sigma_i^{\text{exp}} \pm \Delta\sigma_i^{\text{exp}}$ (mb)	σ_i^{syst} (mb)	Σ_i	EXFOR files
					20527, 20721, 21634, 21936, 22657, 31464, 40226
28	61	62.1 ± 6.6	67.0	0.556	40008, 22637, 30985, 11696, 40136, 20721, 31464, 30979, 31524, 11740, 30811, 40226, 22820
28	62	24.7 ± 2.9	39.0	24.2	40008, 40136, 22156, 22433, 30978, 11696, 20721, 21936, 30811, 40226
29	63	57.6 ± 5.8	77.5	11.8	22418, 13132
29	65	21.8 ± 2.2	26.8	5.19	11474, 21343, 21352, 31449, 40136, 11776, 11536, 30562, 41240, 22089, 22703, 10776, 30263, 20772, 20721, 20888, 21999, 12969, 20799, 11550, 30336, 30707, 31161, 40009, 21419
30	64	171 ± 17	153	1.13	11802, 40016, 22093, 12956, 20748, 20835, 22214, 31449, 13597, 22637, 10224, 11494, 31110, 11515, 30979, 31460, 10022, 10776, 30263, 31500, 20673, 20721, 11740, 13136, 20107, 30336, 30707, 31459, 40009
30	66	72.2 ± 7.2	52.8	7.24	40016, 20280, 20748, 13597, 30642, 22187, 22433, 11515, 11536, 30978, 31460, 10776, 30263, 20673, 20721, 11740, 20887, 30336, 30707, 40009
30	67	49.8 ± 6.0	31.4	9.40	40016, 20748, 13597, 22637, 13044, 30979, 31460, 20673, 40009
30	68	10.9 ± 3.2	18.8	6.12	30978, 22433, 21902, 31460, 21976
31	69	35.8 ± 3.6	36.7	0.0639	21291, 40009
31	71	18.3 ± 1.8	13.4	7.30	20748, 20721, 40009
32	70	91.9 ± 9.2	71.1	5.14	20748, 31434, 40009
32	72	33.7 ± 3.4	26.0	5.16	20748, 11825, 22637, 31434, 20721, 20770, 40009
32	73	22.1 ± 2.2	15.9	8.03	20748, 31491, 11825, 22637, 31434, 22291, 20721, 20770, 40009
32	74	11.4 ± 1.1	9.75	2.25	20748, 11825, 31434, 22291, 20770, 40009
32	76	3.21 ± 0.43	3.75	1.59	22325, 20770
33	75	19.7 ± 2.0	18.7	0.255	20289, 20748, 20898, 11462, 10157, 22637, 20303, 10776, 30263, 22291, 21426, 30336, 30707, 40009

Table 1 (*continued*)

Z	A	$\sigma_i^{exp} \pm \Delta\sigma_i^{exp}$ (mb)	σ_i^{syst} (mb)	Σ_i	EXFOR files
34	74	121 ± 12	94.9	4.72	20034, 20748, 20898, 11722, 41424, 32108, 20721, 31496, 40227, 40009
34	76	53.1 ± 5.3	35.6	10.9	20748, 11722, 41424, 32108, 22291, 20721, 31496, 40227, 40009
34	77	40.0 ± 4.0	22.0	20.3	20748, 32108, 20721, 40009
34	78	19.1 ± 1.9	13.7	8.22	20748, 11722, 41424, 32108, 22291, 20721, 31496, 40227, 40009
35	81	24.0 ± 2.4	10.1	33.5	20107, 40009
37	87	9.18 ± 1.01	5.77	11.4	10157, 22637, 31409, 20811, 20721, 21999, 10088, 21659, 30336
38	84	100.0 ± 10.0	65.3	12.0	20721, 22192
38	86	41.3 ± 4.1	26.1	13.7	40016, 11462, 21300, 40223
38	88	13.2 ± 1.3	10.7	3.84	40016, 22156, 22433, 10157, 21115, 30263, 20721, 21886, 10088, 30336, 30707, 40223
39	89	22.7 ± 3.6	19.6	0.757	11504, 31532, 11462, 31494, 30115, 40223
40	90	44.1 ± 4.4	35.7	3.60	40016, 11896, 11462, 31330, 11856, 20513, 40226
40	91	31.7 ± 3.2	23.0	7.43	40016, 11896, 20513
40	92	19.4 ± 1.9	14.8	5.76	40016, 11896, 30985, 30940, 10145, 41240, 22089, 31330, 20850, 20513, 21976, 31464, 31496, 30336, 40226, 22822, 30438
40	94	6.96 ± 0.70	6.27	0.966	40016, 11896, 30574, 30940, 30985, 22156, 22089, 31330, 20850, 20513, 21976, 31464, 31496, 30336, 30740, 40226, 30438
42	94	56.0 ± 5.6	48.7	1.72	22658, 12976, 31521
42	95	39.9 ± 4.0	31.5	4.36	31491, 32601, 22125, 22089, 41456, 30809, 12976, 21935, 20513, 41321
42	96	23.4 ± 2.3	20.5	1.55	21141, 31491, 30576, 32579, 22125, 10145, 22089, 41424, 41456, 30809, 20850, 21935, 20513, 21999, 31464, 31496, 21840, 41321
42	97	17.2 ± 1.7	13.4	4.93	31491, 10145, 22089, 41456, 30809, 20513, 21999, 21840, 41321
42	98	4.87 ± 1.01	8.81	15.3	31281, 31459, 41321
42	100	2.86 ± 2.14	3.85	0.212	22503, 21840
43	99	13.6 ± 1.4	15.8	2.52	11289, 22655, 41424, 20350, 22817
44	96	140 ± 14	156	1.37	22637, 31437, 11923
44	100	22.9 ± 7.1	28.3	0.570	22325, 30007

(*continued on next page*)

Table 1 (*continued*)

Z	A	$\sigma_i^{\text{exp}} \pm \Delta\sigma_i^{\text{exp}}$ (mb)	σ_i^{syst} (mb)	Σ_i	EXFOR files			
44	101	23.0 ± 3.0	18.6	2.14	22415,	31437		
44	102	1.97 ± 0.83	12.3	155.	11923,	30007		
44	104	6.16 ± 0.65	5.44	1.21	22415,	31437,	11923	
45	103	20.9 ± 2.1	21.9	0.215	10145,	41424		
46	102	90.1 ± 9.0	88.9	0.0164	21609,	32552		
46	105	38.5 ± 3.8	25.7	11.4	32552,	10145,	22637,	30336
46	106	12.2 ± 5.7	17.1	0.736	10145,	40029		
47	109	11.0 ± 2.1	13.5	1.39	40016,	40710,	22100,	11462,
					21426,	30336,	30707,	31161
48	110	20.6 ± 2.1	23.6	2.06	11583,	21976		
48	111	22.7 ± 2.5	15.9	7.44	40016,	11583,	11462,	41424,
					31496			
48	112	16.0 ± 1.6	10.7	10.9	30556,	40016,	11583,	10145,
					22637,	30443,	20540,	31496,
					41424			
48	113	11.6 ± 3.9	7.25	1.24	40016,	11583,	22637	
48	114	7.83 ± 2.56	4.93	1.29	20540,	21976,	30336	
48	116	2.18 ± 0.23	2.29	0.250	20421,	21976		
49	115	11.6 ± 1.2	8.59	6.30	22192,	41240,	40999,	31496,
					40226			
50	112	21.8 ± 9.4	71.6	28.1	40800,	22428,	10484,	30202,
					40798			
50	117	14.5 ± 1.5	10.2	8.39	40800,	10145,	30202,	22089,
					20540,	31496,	30041	
50	118	5.93 ± 0.59	6.94	2.92	20540,	31496		
50	119	6.98 ± 2.16	4.75	1.06	22415,	30202		
52	122	14.2 ± 1.4	12.3	1.87	41240,	20540,	22675,	40029
52	126	3.48 ± 0.35	4.47	8.06	20540,	22675		
52	128	1.67 ± 0.36	2.34	3.42	20540,	11999,	40029	
52	130	0.805 ± 0.467	1.03	0.242	20540,	40029		
54	131	5.94 ± 0.75	4.56	3.37	11884,	10775		
56	136	7.22 ± 0.72	4.65	12.7	22089,	21999		
56	137	8.28 ± 1.95	3.48	6.06	31485,	22205,	31451	
56	138	2.80 ± 0.28	2.53	0.950	30938,	22089,	41240,	32502
57	139	3.87 ± 0.39	4.08	0.302	21289,	22393,	31410,	32205,
					41293,	20509,	30115,	12033,
					12872,	40223		
58	140	7.21 ± 0.72	6.16	2.13	21289,	30777,	22393,	10145,
					41240,	20509,	20716,	12033,
					12872			
58	142	4.44 ± 0.46	3.58	3.52	21289,	20811,	20716,	31497,
					12033,	30336		
59	141	9.64 ± 0.96	8.81	0.740	21289,	22393,	22156,	20509,
					20716,	12033,	12872,	30040
60	142	12.8 ± 1.3	12.1	0.281	21289,	31496,	30649	
60	146	3.56 ± 0.36	4.84	12.7	22348,	22393,	20716,	31496,
					12872,	30649		
62	144	23.4 ± 4.6	20.9	0.305	21609,	20716,	31497	
62	148	11.6 ± 2.2	9.88	0.610	20716,	31497		
62	150	6.39 ± 0.64	6.33	0.00978	20595,	20933,	31497,	12872
63	153	4.96 ± 0.66	5.65	1.09	21213,	22637,	20933,	12872
65	159	5.72 ± 0.57	4.48	4.70	22637,	20509,	20716,	12872

Table 1 (*continued*)

Z	A	$\sigma_i^{exp} \pm \Delta\sigma_i^{exp}$ (mb)	σ_i^{syst} (mb)	Σ_i	EXFOR files
66	160	7.61 ± 0.97	6.51	1.29	20933, 31497
66	162	3.99 ± 0.40	3.99	0.00015	22348, 12131, 30674, 20716, 31497
66	163	3.08 ± 0.31	3.02	0.0319	22348, 12131, 20716, 31497
66	164	2.48 ± 0.27	2.24	0.808	22393, 22340, 20716
68	167	1.50 ± 0.45	4.09	33.1	30471, 31497, 12033, 20860, 30336
68	168	2.58 ± 0.26	3.13	4.43	22393, 20716, 31497, 12033, 20860
70	174	2.86 ± 0.29	2.43	2.19	22348, 30674, 21818, 31497, 12872
70	176	1.76 ± 0.19	1.26	6.97	30674, 22340
71	175	5.08 ± 4.05	3.74	0.109	20595, 20716
72	178	2.54 ± 0.25	3.34	10.1	31439, 13039
72	180	1.94 ± 0.38	1.87	0.0357	22311, 22325
73	181	3.55 ± 0.35	2.96	2.80	12185, 41424, 22351, 32601, 22108, 30671, 32205, 31330
74	182	4.36 ± 0.44	4.41	0.0136	12185, 31484, 41428, 20668, 31496
74	183	4.81 ± 0.48	3.44	8.14	12185, 41428, 20668, 31496
74	184	2.96 ± 0.30	2.63	1.22	31484, 22351, 22089, 41428, 31330 20668 31496
74	186	1.64 ± 0.16	1.42	1.94	22311, 22365, 41428, 31330, 22351, 20668
75	187	4.08 ± 0.45	2.32	15.2	22637, 41240, 20721, 41406
76	188	4.95 ± 0.56	3.55	6.28	22637, 41428, 20198, 30115
76	189	6.83 ± 2.39	2.73	2.95	41428, 20721
77	193	2.89 ± 0.77	1.80	1.99	41428, 20721
80	199	2.87 ± 1.09	2.93	0.00320	12219, 10244
82	208	0.883 ± 0.088	0.899	0.0335	30605, 22148, 10244, 32205, 21504, 40347, 40136, 22680, 22822
83	209	0.780 ± 0.187	1.54	16.6	22148, 40347, 31161

The values of free parameters in Eqs. (8), (11), (12) were obtained from the fitting to experimental cross-sections providing the minimum of the expression

$$\Sigma = \sum_{i=1}^{N} \left(\left(\sigma_i^{syst} - \sigma_i^{exp} \right) / \Delta\sigma_i^{exp} \right)^2, \tag{13}$$

where σ_i^{syst} is the cross-section calculated using Eqs. (8), (11), and (12); σ_i^{exp} and $\Delta\sigma_i^{exp}$ are the cross-section and its error obtained from the analysis of measured data (Section 3.1.1, Table 1); N is the number of target nuclei, for which experimental data are available.

The χ^2 value is calculated as follows

$$\chi^2 = \Sigma / (N - m), \tag{14}$$

where Σ is calculated by Eq. (13) and m is the number of free parameters of the systematics.

Results of the fitting of Eqs. (8), (11), and (12) to experimental cross-sections from Table 1 are given in Table 2.

Table 2
The results of the fitting of Eqs. (8), (11), and (12) to (n, p) reaction cross-sections obtained for 125 target nuclei from the analysis of experimental data at 14.5 MeV (Table 1)

Formula	Σ	χ^2	Parameters
Eq. (8)	830	6.98	$\alpha_1 = -4.4785$, $\alpha_2 = 4.7174 \times 10^{-2}$, $\alpha_3 = -0.27407$, $\alpha_4 = 0.75718$, $\alpha_5 = -0.61348$, $\alpha_6 = 0.1511$
Eq. (12), Konobeyev, Korovin [6]	913	7.73	$\alpha_1 = 2.40$, $\alpha_2 = -0.35257$, $\alpha_3 = 0.10577$, $\alpha_4 = -5.5299 \times 10^{-3}$, $\alpha_5 = 0.62165$, $\alpha_6 = -130.59$, $\alpha_7 = -11.636$
Eq. (11), Belgaid et al. [19]	924	7.77	$\alpha_1 = 1.1279$, $\alpha_2 = -67.502$, $\alpha_3 = -883.55$, $\alpha_4 = 3.4336$, $\alpha_5 = 40.717$, $\alpha_6 = -26.007$

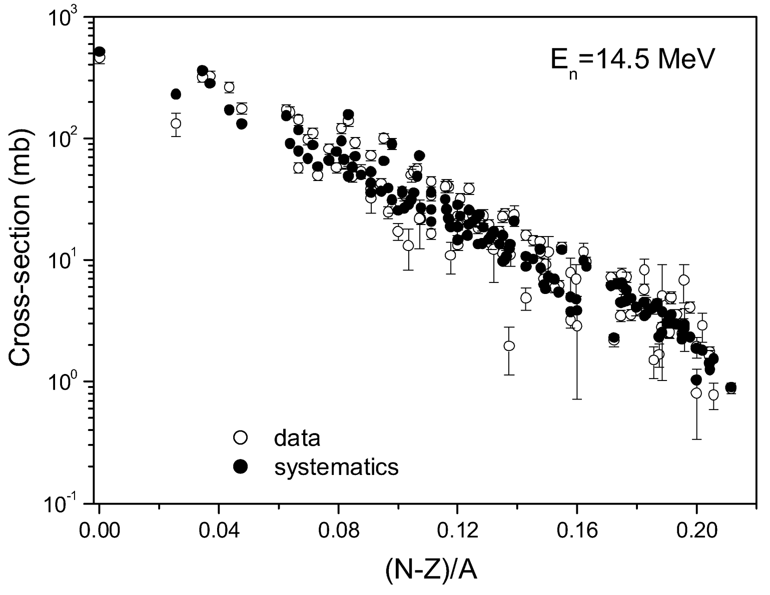

Fig. 1. The (n, p) reaction cross-sections for 125 nuclei with the mass number $39 \leqslant A \leqslant 209$ at the incident neutron energy 14.5 MeV obtained from the analysis of experimental data (Table 1) (open circle) and calculated by Eq. (8) with parameters from Table 2 (black circle) depending upon $(N - Z)/A$.

The comparison of different systematics shows that the formula, Eq. (8) proposed in the present work provides the best fit of experimental data. Fig. 1 shows cross-sections evaluated by Eq. (8) and experimental cross-sections from Table 1.

Table 3 shows values of other statistical factors [20], which characterize the deviation of cross-sections calculated using systematic formulas, Eqs. (8), (11), (12) from experimental data. Compared with χ^2 (Table 2) the deviation factors from Table 3 are of secondary importance.

It should be noted that the division of nuclei on two groups with atomic number below and above 50 with the predominant contribution of the equilibrium and precompound emission (Section 2.3) is in a general agreement with calculations performed using geometry dependent hybrid precompound model and evaporation model [13,21]. The use of other critical value of atomic number, Z_{crit} for the transition from Eq. (8a) to Eq. (8b) changes smoothly the value of Σ at the fitting of systematic formulas to experimental data from Table 1. For the transition from Eq. (8a) to Eq. (8b) at Z_{crit} equal to 46 the Σ value is equal to 863, at $Z_{\mathrm{crit}} = 48$ the Σ value is 819, for

Table 3
Deviation factors [20] calculated using cross-sections predicted by various systematics Eqs. (8), (11), and (12) and experimental cross-sections from Table 1

Factors	Eq. (8)	Eq. (12), Konobeyev, Korovin [6]	Eq. (11), Belgaid et al. [19]
$N^{-1}\sum_{i=1}^{N}\lvert\sigma_i^{\mathrm{exp}}-\sigma_i^{\mathrm{syst}}\rvert/\sigma_i^{\mathrm{exp}}$	0.317	0.319	0.325
$N^{-1}\sum_{i=1}^{N}\sigma_i^{\mathrm{syst}}/\sigma_i^{\mathrm{exp}}$	1.026	1.018	1.023
$10^{(N^{-1}\sum_{i=1}^{N}[\log(\sigma_i^{\mathrm{exp}})-\log(\sigma_i^{\mathrm{syst}})]^2)^{0.5}}$	1.466	1.466	1.471

$Z_{\mathrm{crit}}=50$ one gets $\Sigma=830$ (Table 2), Z_{crit} equal to 52 corresponds to $\Sigma=833$, and $Z_{\mathrm{crit}}=54$ gives the Σ value equal to 854.

3.2. Incident neutrons of energy 20 MeV

At present time the systematics of neutron induced reaction cross-sections at 20 MeV are used for the verification and validation of evaluated activation data [22]. The first systematics at 20 MeV has been obtained in Ref. [23] using results of model calculations with the global set of model parameters. In this work the method [23] is noticeably improved.

The experimental data at 20 MeV are too scarce to be a reliable foundation of the systematics. In the present work Eq. (8) was fitted to the data at 20 MeV obtained with the help of model calculations corrected using the available experimental information.

The calculation of the (n,p) reaction cross-section at 20 MeV was performed using a preequilibrium geometry dependent hybrid model [13] and an evaporation model [14] implemented in the ALICE/ASH code [21]. The parameters of models were selected from the condition of the accurate description of experimental (n,p) reaction cross-sections at 14.5 MeV (Table 1). The obtained model parameters were used to calculate the (n,p) reaction cross-section at 20 MeV and 30 MeV (Section 3.3). The calculated cross-sections (σ^{eval}) are shown in Fig. 2 for 125 nuclei.

The systematics was obtained by minimization of Σ the value, Eq. (13), where σ^{eval} values are used instead of σ^{exp} and σ^{syst} is calculated by Eq. (8). The obtained parameters of Eq. (8) are $\alpha_1=-1.4772$, $\alpha_2=4.2553\times10^{-2}$, $\alpha_3=-0.65261$, $\alpha_4=-0.71595$, $\alpha_5=-5.1645$, $\alpha_6=1.5205$.

Fig. 3 shows the example of the (n,p) reaction cross-section calculated using the systematics.

3.3. Incident neutrons of energy 30 MeV

The aim is to get an additional tool for the evaluation and verification of (n,p) reaction cross-sections at intermediate energies. It seems important due to the intention to extend commonly used evaluated data files up to 60 MeV [22] and up to 150 MeV [24]. A neutron energy of 30 MeV is a reasonable limit for the systematics considering that the uncertainty of σ^{eval} values used for the fitting of free parameters, obtained as described in Section 3.2, increases with the energy of neutrons.

At the transition to 30 MeV the importance of the preequilibrium single-proton emission increases. To estimate the (n,p) reaction cross-section one should consider the possible particle emission from the $(Z-1,A)$ nucleus formed after the escape of the first proton. Assuming that the neutron emission from this nucleus is the most probable, after the integration of Eq. (2) from

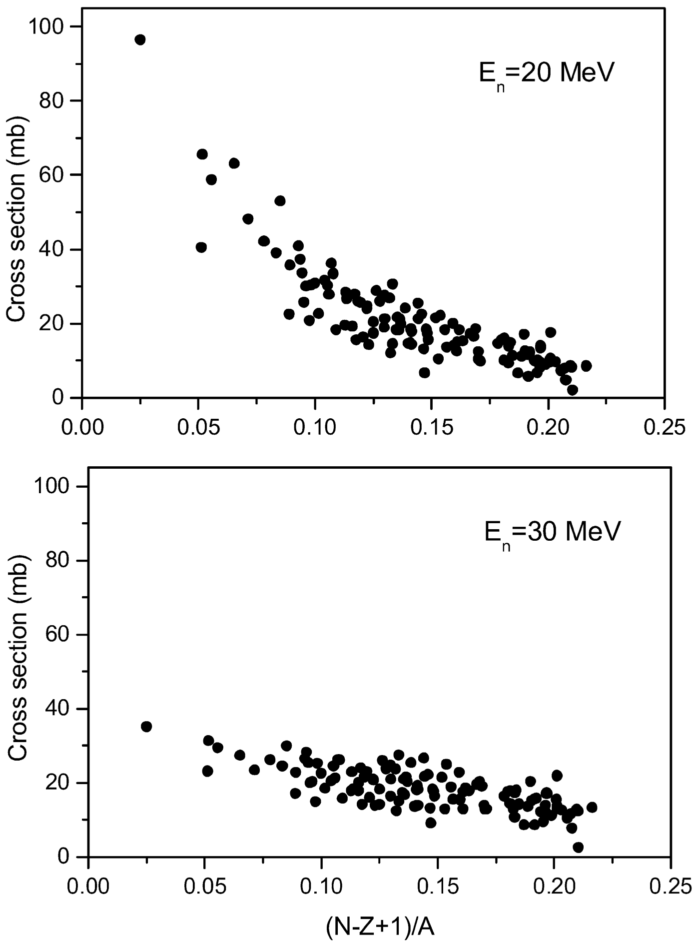

Fig. 2. Cross-sections of the $(n,\,p)$ reaction induced by 20 MeV and 30 MeV calculated for 125 nuclei with the mass number $39 < A < 209$ using the preequilibrium model and the evaporation model implemented in the ALICE/ASH code [21]. The parameters of models have been obtained from the condition of the accurate description of the experimental $(n,\,p)$ reaction cross-sections at 14.5 MeV (Table 1).

the energy $\alpha E_n + Q_{(n,p)} - Q'_n$ to $\alpha E_n + Q_{(n,p)}$ the approximate expression for the precompound component of the $(n,\,p)$ reaction cross-section can be written in the following form

$$\sigma^{\text{pre}}_{(n,p)} = \sigma_{\text{non}}(E_n)\frac{(2S_p+1)\mu_p R^2}{\pi^2\hbar^2 g^4 E_0^4|M|^2}\big[(\alpha E_n + Q_{(n,p)} - V_p)^3 - (\alpha E_n + Q_{(n,p)} - V_p - Q'_n)^2(\alpha E_n + Q_{(n,p)} - V_p + 2Q'_n)\big], \tag{15}$$

where the Q'_n is the separation energy of neutron from the nucleus $(Z-1,\,A)$, which can be approximately calculated as

$$Q'_n = \gamma'_1\left(\frac{N-Z+1}{A}\right)^2 + \gamma'_2\left(\frac{N-Z+1.5}{A}\right) + \gamma'_3 Z^2/A^{4/3} + \gamma'_4/A^{1/3} + \gamma'_5, \tag{16}$$

where γ'_i are constants.

It is seen that the cross-section, Eq. (15) is expressed by polynomial functions of $Q_{(n,p)}$ and Q'_n. Taking into account the essential dependence of these values from $N - Z + 1/A$, the following x-parametric formula is suggested for the systematics at 30 MeV

$$\sigma_{(n,p)} = \pi r_0^2 \left(A^{1/3} + 1\right)^2 A^{\alpha_1} \left(\alpha_2 S^3 + \alpha_3 S^2 + \alpha_4 S + \alpha_5 Z/A^{1/3} + \alpha_6\right), \tag{17}$$

where $S = (N - Z + 1)/A$, $P = (N - Z + 1.5)/A$, $r_0 = 1.3$ fm, α_i are parameters.

The fitting of Eq. (17) to the σ^{eval} values obtained at 30 MeV for 125 nuclei (Fig. 2) gives the following values of parameters: $\alpha_1 = 0.33239$, $\alpha_2 = -2.1417$, $\alpha_3 = 1.0792$, $\alpha_4 = -0.19660$, $\alpha_5 = -1.7282 \times 10^{-4}$, $\alpha_6 = 1.6253 \times 10^{-2}$.

Fig. 3 shows the (n, p) reaction cross-section evaluated using the systematics for ^{51}V and ^{56}Fe.

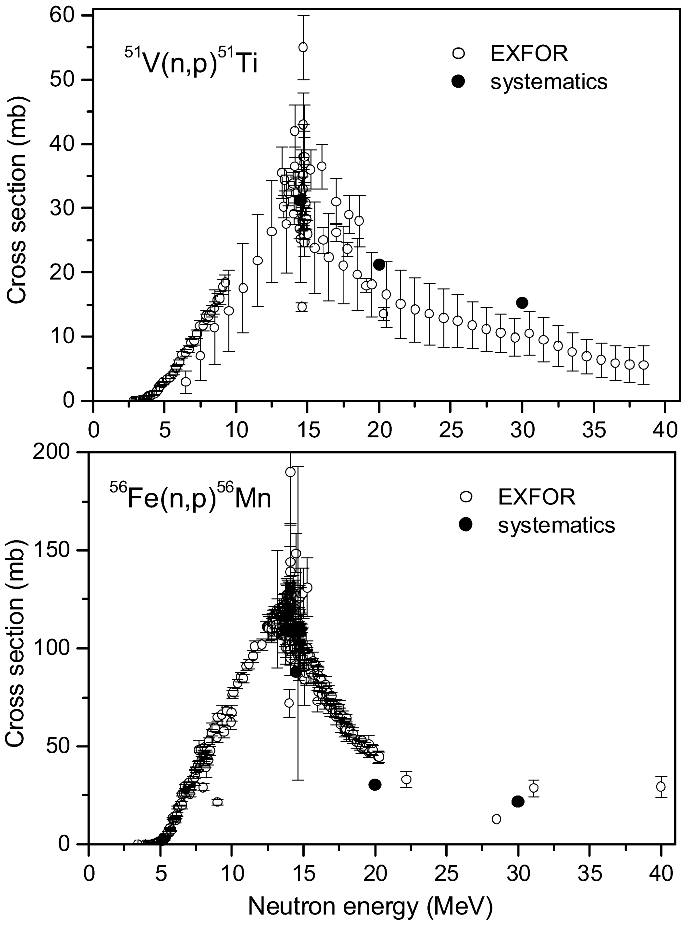

Fig. 3. The (n, p) reaction cross-section for ^{51}V and ^{56}Fe evaluated by the systematics obtained in the present work (black circle) and experimental data from EXFOR (open circle).

4. Conclusion

A new systematics for the (n, p) reaction cross-section at the incident neutron energy 14.5, 20, and 30 MeV is suggested. The preequilibrium exciton model, the evaporation model and a semi-empirical mass formula were used to get analytical expressions for the evaluation of the cross-section.

A new compilation and analysis of experimental (n, p) reaction cross-section at neutron incident energies 14–15 MeV has been performed. The comparison of different systematics shows that the formula proposed in the present work gives the best description of experimental data.

The obtained systematics are for 14.5 MeV incident neutrons:

$$\sigma_{(n,p)} = \pi r_0^2 \left(A^{1/3}+1\right)^2 \exp\left(A^{0.5}\left(-4.4785S + 4.7174 \times 10^{-2} V - 0.27407\right)\right) \quad \text{if } Z \leqslant 50, \tag{18a}$$

$$\sigma_{(n,p)} = \pi r_0^2 \left(A^{1/3}+1\right)^2 A^{0.75718} (-0.61348S + 0.1511)^3 \quad \text{if } Z > 50, \tag{18b}$$

for 20 MeV neutrons:

$$\sigma_{(n,p)} = \pi r_0^2 \left(A^{1/3}+1\right)^2 \exp\left(A^{0.5}\left(-1.4772S + 4.2553 \times 10^{-2} - 0.65261\right)\right) \quad \text{if } Z \leqslant 50, \tag{19a}$$

$$\sigma_{(n,p)} = \pi r_0^2 \left(A^{1/3}+1\right)^2 A^{-0.71595} (-5.1645S + 1.5205)^3 \quad \text{if } Z > 50, \tag{19b}$$

for 30 MeV neutrons:

$$\sigma_{(n,p)} = \pi r_0^2 \left(A^{1/3}+1\right)^2 A^{0.33239}\left(-2.1417S^3 + 1.0792S^2 - 0.19660S - 1.7282 \times 10^{-4} + 1.6253 \times 10^{-2}\right), \tag{20}$$

where $S = (N - Z + 1)/A$, $V = Z/A^{1/3}$; r_0 is equal to 1.3 fm, which corresponds to the πr_0^2 value equal to 53.093 mb; N, Z, and A are the number of neutrons, protons, and nucleons in the target nucleus, correspondingly.

The systematics obtained can be used for the evaluation of the (n, p) reaction cross-section for nuclei $A \geqslant 40$. The best result one should expect for the range of target nuclei $0.025 < (N - Z)/A < 0.22$. Outside of these limits the systematics, Eqs. (18)–(20) should be used with care.

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